



NINTH EDITION

BIRD'S HIGHER ENGINEERING MATHEMATICS

JOHN BIRD

Bird's Higher Engineering Mathematics

Why is knowledge of mathematics important in engineering?

A career in any engineering or scientific field will require both basic and advanced mathematics. Without mathematics to determine principles, calculate dimensions and limits, explore variations, prove concepts and so on, there would be no mobile telephones, televisions, stereo systems, video games, microwave ovens, computers or virtually anything electronic. There would be no bridges, tunnels, roads, skyscrapers, automobiles, ships, planes, rockets or most things mechanical. There would be no metals beyond the common ones, such as iron and copper, no plastics, no synthetics. In fact, society would most certainly be less advanced without the use of mathematics throughout the centuries and into the future.

Electrical engineers require mathematics to design, develop, test or supervise the manufacturing and installation of electrical equipment, components or systems for commercial, industrial, military or scientific use.

Mechanical engineers require mathematics to perform engineering duties in planning and designing tools, engines, machines and other mechanically functioning equipment; they oversee installation, operation, maintenance and repair of such equipment as centralised heat, gas, water and steam systems.

Aerospace engineers require mathematics to perform a variety of engineering work in designing, constructing and testing aircraft, missiles and spacecraft; they conduct basic and applied research to evaluate adaptability of materials and equipment to aircraft design and manufacture and recommend improvements in testing equipment and techniques.

Nuclear engineers require mathematics to conduct research on nuclear engineering problems or apply

principles and theory of nuclear science to problems concerned with release, control and utilisation of nuclear energy and nuclear waste disposal.

Petroleum engineers require mathematics to devise methods to improve oil and gas well production and determine the need for new or modified tool designs; they oversee drilling and offer technical advice to achieve economical and satisfactory progress.

Industrial engineers require mathematics to design, develop, test and evaluate integrated systems for managing industrial production processes, including human work factors, quality control, inventory control, logistics and material flow, cost analysis and production co-ordination.

Environmental engineers require mathematics to design, plan or perform engineering duties in the prevention, control and remediation of environmental health hazards, using various engineering disciplines; their work may include waste treatment, site remediation or pollution control technology.

Civil engineers require mathematics in all levels in civil engineering – structural engineering, hydraulics and geotechnical engineering are all fields that employ mathematical tools such as differential equations, tensor analysis, field theory, numerical methods and operations research.

Knowledge of mathematics is therefore needed by each of the engineering disciplines listed above.

It is intended that this text – *Bird's Higher Engineering Mathematics* – will provide a step-by-step approach to learning the essential mathematics needed for your engineering studies.

Now in its ninth edition, *Bird's Higher Engineering Mathematics* has helped thousands of students to succeed in their exams. Mathematical theories are explained in a straightforward manner, supported by practical engineering examples and applications to ensure that readers can relate theory to practice. Some 1,200 engineering situations/problems have been 'flagged-up' to help demonstrate that engineering cannot be fully understood without a good knowledge of mathematics.

The extensive and thorough topic coverage makes this an ideal text for undergraduate degree courses, foundation degrees, and for higher-level vocational courses such as Higher National Certificate and Diploma courses in engineering disciplines.

Its companion website at www.routledge.com/cw/bird provides resources for both students and lecturers, including full solutions for all 2,100 further questions, lists of essential formulae, multiple-choice tests, and illustrations, as well as full solutions to revision tests for course instructors.

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To Sue

Contents

Preface	xiv	5.4 Further worked problems on the binomial series	51
Syllabus guidance	xvi	5.5 Practical problems involving the binomial theorem	54
Section A Number and algebra		1	
1 Algebra	3	6 Solving equations by iterative methods	57
1.1 Introduction	3	6.1 Introduction to iterative methods	57
1.2 Revision of basic laws	3	6.2 The bisection method	58
1.3 Revision of equations	5	6.3 An algebraic method of successive approximations	61
1.4 Polynomial division	9	7 Boolean algebra and logic circuits	65
1.5 The factor theorem	11	7.1 Boolean algebra and switching circuits	66
1.6 The remainder theorem	13	7.2 Simplifying Boolean expressions	70
2 Partial fractions	17	7.3 Laws and rules of Boolean algebra	70
2.1 Introduction to partial fractions	17	7.4 De Morgan's laws	72
2.2 Partial fractions with linear factors	17	7.5 Karnaugh maps	73
2.3 Partial fractions with repeated linear factors	20	7.6 Logic circuits	77
2.4 Partial fractions with quadratic factors	22	7.7 Universal logic gates	81
3 Logarithms	24	Revision Test 2 85	
3.1 Introduction to logarithms	24	Section B Geometry and trigonometry 87	
3.2 Laws of logarithms	26	8 Introduction to trigonometry	89
3.3 Indicial equations	28	8.1 Trigonometry	90
3.4 Graphs of logarithmic functions	30	8.2 The theorem of Pythagoras	90
4 Exponential functions	32	8.3 Trigonometric ratios of acute angles	91
4.1 Introduction to exponential functions	32	8.4 Evaluating trigonometric ratios	93
4.2 The power series for e^x	33	8.5 Solution of right-angled triangles	97
4.3 Graphs of exponential functions	35	8.6 Angles of elevation and depression	99
4.4 Napierian logarithms	36	8.7 Sine and cosine rules	100
4.5 Laws of growth and decay	40	8.8 Area of any triangle	101
4.6 Reduction of exponential laws to linear form	43	8.9 Worked problems on the solution of triangles and finding their areas	101
Revision Test 1 47		8.10 Further worked problems on solving triangles and finding their areas	102
5 The binomial series	48	8.11 Practical situations involving trigonometry	104
5.1 Pascal's triangle	48	8.12 Further practical situations involving trigonometry	106
5.2 The binomial series	50		
5.3 Worked problems on the binomial series	50		

9 Cartesian and polar co-ordinates	111	14 The relationship between trigonometric and hyperbolic functions	169
9.1 Introduction	112	14.1 The relationship between trigonometric and hyperbolic functions	169
9.2 Changing from Cartesian into polar co-ordinates	112	14.2 Hyperbolic identities	170
9.3 Changing from polar into Cartesian co-ordinates	114	15 Compound angles	173
9.4 Use of Pol/Rec functions on calculators	115	15.1 Compound angle formulae	173
10 The circle and its properties	117	15.2 Conversion of a $\sin \omega t + b \cos \omega t$ into $R \sin(\omega t + \alpha)$	175
10.1 Introduction	117	15.3 Double angles	179
10.2 Properties of circles	117	15.4 Changing products of sines and cosines into sums or differences	181
10.3 Radians and degrees	119	15.5 Changing sums or differences of sines and cosines into products	182
10.4 Arc length and area of circles and sectors	120	15.6 Power waveforms in a.c. circuits	183
10.5 The equation of a circle	123		
10.6 Linear and angular velocity	125		
10.7 Centripetal force	126		
Revision Test 3	130	Revision Test 4	187
11 Trigonometric waveforms	132	Section C Graphs	189
11.1 Graphs of trigonometric functions	132		
11.2 Angles of any magnitude	133	16 Functions and their curves	191
11.3 The production of a sine and cosine wave	136	16.1 Standard curves	191
11.4 Sine and cosine curves	137	16.2 Simple transformations	194
11.5 Sinusoidal form $A \sin(\omega t \pm \alpha)$	141	16.3 Periodic functions	199
11.6 Harmonic synthesis with complex waveforms	144	16.4 Continuous and discontinuous functions	199
		16.5 Even and odd functions	200
		16.6 Inverse functions	201
		16.7 Asymptotes	203
		16.8 Brief guide to curve sketching	208
		16.9 Worked problems on curve sketching	208
12 Hyperbolic functions	151	17 Irregular areas, volumes and mean values of waveforms	213
12.1 Introduction to hyperbolic functions	151	17.1 Areas of irregular figures	213
12.2 Graphs of hyperbolic functions	153	17.2 Volumes of irregular solids	216
12.3 Hyperbolic identities	155	17.3 The mean or average value of a waveform	217
12.4 Solving equations involving hyperbolic functions	157		
12.5 Series expansions for $\cosh x$ and $\sinh x$	159		
13 Trigonometric identities and equations	161	Revision Test 5	223
13.1 Trigonometric identities	161		
13.2 Worked problems on trigonometric identities	162	Section D Complex numbers	225
13.3 Trigonometric equations	163		
13.4 Worked problems (i) on trigonometric equations	164	18 Complex numbers	227
13.5 Worked problems (ii) on trigonometric equations	165	18.1 Cartesian complex numbers	228
13.6 Worked problems (iii) on trigonometric equations	166	18.2 The Argand diagram	229
13.7 Worked problems (iv) on trigonometric equations	166	18.3 Addition and subtraction of complex numbers	229

18.4	Multiplication and division of complex numbers	230	22.5	Resolving vectors into horizontal and vertical components	289
18.5	Complex equations	232	22.6	Addition of vectors by calculation	290
18.6	The polar form of a complex number	233	22.7	Vector subtraction	295
18.7	Multiplication and division in polar form	235	22.8	Relative velocity	297
18.8	Applications of complex numbers	236	22.9	i, j and k notation	298
19	De Moivre's theorem	241	23	Methods of adding alternating waveforms	301
19.1	Introduction	242	23.1	Combination of two periodic functions	301
19.2	Powers of complex numbers	242	23.2	Plotting periodic functions	302
19.3	Roots of complex numbers	243	23.3	Determining resultant phasors by drawing	303
19.4	The exponential form of a complex number	245	23.4	Determining resultant phasors by the sine and cosine rules	305
19.5	Introduction to locus problems	246	23.5	Determining resultant phasors by horizontal and vertical components	306
Section E Matrices and determinants		251	23.6	Determining resultant phasors by using complex numbers	308
20	The theory of matrices and determinants	253	24	Scalar and vector products	312
20.1	Matrix notation	253	24.1	The unit triad	312
20.2	Addition, subtraction and multiplication of matrices	254	24.2	The scalar product of two vectors	313
20.3	The unit matrix	257	24.3	Vector products	317
20.4	The determinant of a 2 by 2 matrix	257	24.4	Vector equation of a line	321
20.5	The inverse or reciprocal of a 2 by 2 matrix	258	Revision Test 7 323		
20.6	The determinant of a 3 by 3 matrix	259	Section G Differential calculus 325		
20.7	The inverse or reciprocal of a 3 by 3 matrix	261	25	Methods of differentiation	327
21	Applications of matrices and determinants	265	25.1	Introduction to calculus	327
21.1	Solution of simultaneous equations by matrices	266	25.2	The gradient of a curve	327
21.2	Solution of simultaneous equations by determinants	268	25.3	Differentiation from first principles	328
21.3	Solution of simultaneous equations using Cramer's rule	271	25.4	Differentiation of common functions	329
21.4	Solution of simultaneous equations using the Gaussian elimination method	272	25.5	Differentiation of a product	332
21.5	Stiffness matrix	274	25.6	Differentiation of a quotient	334
21.6	Eigenvalues and eigenvectors	275	25.7	Function of a function	335
Revision Test 6 281			25.8	Successive differentiation	337
Section F Vector geometry 283			26	Some applications of differentiation	340
22	Vectors	285	26.1	Rates of change	340
22.1	Introduction	285	26.2	Velocity and acceleration	342
22.2	Scalars and vectors	285	26.3	The Newton–Raphson method	345
22.3	Drawing a vector	286	26.4	Turning points	348
22.4	Addition of vectors by drawing	287	26.5	Practical problems involving maximum and minimum values	351
			26.6	Points of inflexion	355
			26.7	Tangents and normals	357
			26.8	Small changes	358
Revision Test 8 362					

39 Integration using trigonometric and hyperbolic substitutions	467	44 Double and triple integrals	506
39.1 Introduction	467	44.1 Double integrals	506
39.2 Worked problems on integration of $\sin^2 x$, $\cos^2 x$, $\tan^2 x$ and $\cot^2 x$	467	44.2 Triple integrals	508
39.3 Worked problems on integration of powers of sines and cosines	470	45 Numerical integration	512
39.4 Worked problems on integration of products of sines and cosines	471	45.1 Introduction	512
39.5 Worked problems on integration using the $\sin \theta$ substitution	472	45.2 The trapezoidal rule	512
39.6 Worked problems on integration using the $\tan \theta$ substitution	474	45.3 The mid-ordinate rule	515
39.7 Worked problems on integration using the $\sinh \theta$ substitution	474	45.4 Simpson's rule	516
39.8 Worked problems on integration using the $\cosh \theta$ substitution	476	45.5 Accuracy of numerical integration	520
40 Integration using partial fractions	479	Revision Test 13	521
40.1 Introduction	479	Section I Differential equations	523
40.2 Integration using partial fractions with linear factors	479	46 Introduction to differential equations	525
40.3 Integration using partial fractions with repeated linear factors	481	46.1 Family of curves	525
40.4 Integration using partial fractions with quadratic factors	482	46.2 Differential equations	526
41 The $t = \tan \frac{\theta}{2}$ substitution	485	46.3 The solution of equations of the form $\frac{dy}{dx} = f(x)$	527
41.1 Introduction	485	46.4 The solution of equations of the form $\frac{dy}{dx} = f(y)$	528
41.2 Worked problems on the $t = \tan \frac{\theta}{2}$ substitution	486	46.5 The solution of equations of the form $\frac{dy}{dx} = f(x)f(y)$	530
41.3 Further worked problems on the $t = \tan \frac{\theta}{2}$ substitution	487	47 Homogeneous first-order differential equations	534
Revision Test 12	490	47.1 Introduction	534
42 Integration by parts	491	47.2 Procedure to solve differential equations of the form $P \frac{dy}{dx} = Q$	534
42.1 Introduction	491	47.3 Worked problems on homogeneous first-order differential equations	535
42.2 Worked problems on integration by parts	491	47.4 Further worked problems on homogeneous first-order differential equations	536
42.3 Further worked problems on integration by parts	493	48 Linear first-order differential equations	538
43 Reduction formulae	497	48.1 Introduction	538
43.1 Introduction	497	48.2 Procedure to solve differential equations of the form $\frac{dy}{dx} + Py = Q$	539
43.2 Using reduction formulae for integrals of the form $\int x^n e^x dx$	497	48.3 Worked problems on linear first-order differential equations	539
43.3 Using reduction formulae for integrals of the form $\int x^n \cos x dx$ and $\int x^n \sin x dx$	498	48.4 Further worked problems on linear first-order differential equations	540
43.4 Using reduction formulae for integrals of the form $\int \sin^n x dx$ and $\int \cos^n x dx$	501	49 Numerical methods for first-order differential equations	543
43.5 Further reduction formulae	503	49.1 Introduction	543
		49.2 Euler's method	544
		49.3 Worked problems on Euler's method	545

49.4	The Euler–Cauchy method	549	52.5	Power series solution by the Frobenius method	585
49.5	The Runge–Kutta method	554	52.6	Bessel’s equation and Bessel’s functions	592
Revision Test 14		560	52.7	Legendre’s equation and Legendre polynomials	597
50	Second-order differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$	561	53	An introduction to partial differential equations	601
50.1	Introduction	561	53.1	Introduction	602
50.2	Procedure to solve differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$	562	53.2	Partial integration	602
50.3	Worked problems on differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$	562	53.3	Solution of partial differential equations by direct partial integration	602
50.4	Further worked problems on practical differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$	564	53.4	Some important engineering partial differential equations	604
51	Second-order differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$	568	53.5	Separating the variables	605
51.1	Complementary function and particular integral	569	53.6	The wave equation	606
51.2	Procedure to solve differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$	570	53.7	The heat conduction equation	610
51.3	Differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ where $f(x)$ is a constant or polynomial	570	53.8	Laplace’s equation	612
51.4	Differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ where $f(x)$ is an exponential function	571	Revision Test 15		615
51.5	Differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ where $f(x)$ is a sine or cosine function	573	Section J		Laplace transforms
51.6	Differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ where $f(x)$ is a sum or a product	575			617
52	Power series methods of solving ordinary differential equations	578	54	Introduction to Laplace transforms	619
52.1	Introduction	578	54.1	Introduction	619
52.2	Higher order differential coefficients as series	579	54.2	Definition of a Laplace transform	619
52.3	Leibniz’s theorem	580	54.3	Linearity property of the Laplace transform	620
52.4	Power series solution by the Leibniz-Maclaurin method	583	54.4	Laplace transforms of elementary functions	620
			54.5	Worked problems on standard Laplace transforms	621
			55	Properties of Laplace transforms	625
			55.1	The Laplace transform of $e^{at}f(t)$	625
			55.2	Laplace transforms of the form $e^{at}f(t)$	625
			55.3	The Laplace transforms of derivatives	627
			55.4	The initial and final value theorems	629
			56	Inverse Laplace transforms	632
			56.1	Definition of the inverse Laplace transform	632
			56.2	Inverse Laplace transforms of simple functions	632
			56.3	Inverse Laplace transforms using partial fractions	635
			56.4	Poles and zeros	637
			57	The Laplace transform of the Heaviside function	640
			57.1	Heaviside unit step function	640
			57.2	Laplace transforms of $H(t - c)$	644
			57.3	Laplace transforms of $H(t - c).f(t - c)$	644
			57.4	Inverse Laplace transforms of Heaviside functions	645

58 The solution of differential equations using Laplace transforms	648
58.1 Introduction	648
58.2 Procedure to solve differential equations using Laplace transforms	648
58.3 Worked problems on solving differential equations using Laplace transforms	649

59 The solution of simultaneous differential equations using Laplace transforms	653
59.1 Introduction	653
59.2 Procedure to solve simultaneous differential equations using Laplace transforms	653
59.3 Worked problems on solving simultaneous differential equations using Laplace transforms	654

Revision Test 16	659
-------------------------	------------

Section K Fourier series	661
---------------------------------	------------

60 Fourier series for periodic functions of period 2π	663
60.1 Introduction	664
60.2 Periodic functions	664
60.3 Fourier series	664
60.4 Worked problems on Fourier series of periodic functions of period 2π	665

61 Fourier series for a non-periodic function over period 2π	670
61.1 Expansion of non-periodic functions	670
61.2 Worked problems on Fourier series of non-periodic functions over a range of 2π	671

62 Even and odd functions and half-range Fourier series	676
62.1 Even and odd functions	676
62.2 Fourier cosine and Fourier sine series	676
62.3 Half-range Fourier series	680

63 Fourier series over any range	684
63.1 Expansion of a periodic function of period L	684
63.2 Half-range Fourier series for functions defined over range L	688

64 A numerical method of harmonic analysis	690
64.1 Introduction	690
64.2 Harmonic analysis on data given in tabular or graphical form	690
64.3 Complex waveform considerations	694

65 The complex or exponential form of a Fourier series	698
65.1 Introduction	698
65.2 Exponential or complex notation	698
65.3 Complex coefficients	699
65.4 Symmetry relationships	703
65.5 The frequency spectrum	706
65.6 Phasors	707

Section L Z-transforms	711
-------------------------------	------------

66 An introduction to z-transforms	713
66.1 Sequences	714
66.2 Some properties of z-transforms	717
66.3 Inverse z-transforms	720
66.4 Using z-transforms to solve difference equations	722

Revision Test 17	727
-------------------------	------------

Section M Statistics and probability	729
---	------------

67 Presentation of statistical data	731
67.1 Some statistical terminology	732
67.2 Presentation of ungrouped data	733
67.3 Presentation of grouped data	736

68 Mean, median, mode and standard deviation	743
68.1 Measures of central tendency	743
68.2 Mean, median and mode for discrete data	744
68.3 Mean, median and mode for grouped data	745
68.4 Standard deviation	746
68.5 Quartiles, deciles and percentiles	748

69 Probability	751
69.1 Introduction to probability	752
69.2 Laws of probability	752
69.3 Worked problems on probability	753
69.4 Further worked problems on probability	755
69.5 Permutations and combinations	758
69.6 Bayes' theorem	759

Revision Test 18	762
-------------------------	------------

70 The binomial and Poisson distributions	764
70.1 The binomial distribution	764
70.2 The Poisson distribution	767

71 The normal distribution	771	75 Significance testing	805
71.1 Introduction to the normal distribution	771	75.1 Hypotheses	805
71.2 Testing for a normal distribution	776	75.2 Type I and type II errors	806
72 Linear correlation	780	75.3 Significance tests for population means	812
72.1 Introduction to linear correlation	780	75.4 Comparing two sample means	817
72.2 The Pearson product-moment formula for determining the linear correlation coefficient	780	76 Chi-square and distribution-free tests	822
72.3 The significance of a coefficient of correlation	781	76.1 Chi-square values	822
72.4 Worked problems on linear correlation	781	76.2 Fitting data to theoretical distributions	824
73 Linear regression	785	76.3 Introduction to distribution-free tests	830
73.1 Introduction to linear regression	785	76.4 The sign test	830
73.2 The least-squares regression lines	785	76.5 Wilcoxon signed-rank test	833
73.3 Worked problems on linear regression	786	76.6 The Mann-Whitney test	837
		Revision Test 20	844
Revision Test 19	791	Essential formulae	846
74 Sampling and estimation theories	792	Answers to Practice Exercises	863
74.1 Introduction	792	Index	910
74.2 Sampling distributions	792		
74.3 The sampling distribution of the means	793		
74.4 The estimation of population parameters based on a large sample size	796		
74.5 Estimating the mean of a population based on a small sample size	801		

Preface

This **ninth edition of 'Bird's Higher Engineering Mathematics'** covers essential mathematical material suitable for students studying **Degrees, Foundation Degrees and Higher National Certificate and Diploma courses in Engineering disciplines.**

The text has been conveniently divided into the following **thirteen convenient categories**: number and algebra, geometry and trigonometry, graphs, complex numbers, matrices and determinants, vector geometry, differential calculus, integral calculus, differential equations, Laplace transforms, Fourier series, Z-transforms and statistics and probability.

Increasingly, **difficulty in understanding algebra** is proving a problem for many students as they commence studying engineering courses. Inevitably there are a lot of formulae and calculations involved with engineering studies that require a sound grasp of algebra. On the website, www.routledge.com/bird is a document which offers **a quick revision of the main areas of algebra** essential for further study, i.e. basic algebra, simple equations, transposition of formulae, simultaneous equations and quadratic equations.

In this new edition, all but three of the chapters of the previous edition are included (those excluded can be found in *Bird's Engineering Mathematics 8th Edition* or on the website), but the order of presenting some of the chapters has been changed. Problems where **engineering applications** occur have been 'flagged up' and some multiple-choice questions have been added to many of the chapters.

The **primary aim of the material in this text** is to provide the fundamental analytical and underpinning knowledge and techniques needed to successfully complete scientific and engineering principles modules of Degree, Foundation Degree and Higher National Engineering programmes. The material has been designed to enable students to use techniques learned for the analysis, modelling and solution of realistic engineering problems at Degree and Higher National level. It also aims to provide some of the more advanced knowledge

required for those wishing to pursue careers in mechanical engineering, aeronautical engineering, electrical and electronic engineering, communications engineering, systems engineering and all variants of control engineering.

In *Bird's Higher Engineering Mathematics 9th Edition*, theory is introduced in each chapter by a full outline of essential definitions, formulae, laws, procedures etc; **problem solving** is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then through solving similar problems themselves.


Access to the plethora of **software packages**, or a graphics calculator, will enhance understanding of some of the topics in this text.

Each topic considered in the text is presented in a way that assumes in the reader only knowledge attained in BTEC National Certificate/Diploma, or similar, in an Engineering discipline.

'Bird's Higher Engineering Mathematics 9th Edition' provides a follow-up to 'Bird's Engineering Mathematics 9th Edition'.

This textbook contains over **1100 worked problems**, followed by some **2100 further problems (with answers)**, arranged within **317 Practice Exercises**. Some **450 multiple-choice questions** are also included, together with **573 line diagrams** to further enhance understanding.

Worked solutions to all 2100 of the further problems have been prepared and can be **accessed free by students and staff via the website www.routledge.com/bird**

Where at all possible, the problems mirror practical situations found in engineering and science. In fact, some **1200 engineering situations/problems** have been 'flagged-up' to help demonstrate that engineering cannot be fully understood without a good knowledge of mathematics. Look out for the symbol 

At the end of the text, a list of **Essential Formulae** is included for convenience of reference.

At intervals throughout the text are some **20 Revision Tests** to check understanding. For example, Revision Test 1 covers the material in [chapters 1 to 4](#), Revision Test 2 covers the material in [chapters 5 to 7](#), Revision Test 3 covers the material in [chapters 8 to 10](#), and so on. Full solutions to the 20 Revision Tests are available free to lecturers on the website www.routledge.com/cw/bird

‘Learning by example’ is at the heart of ‘Bird’s Higher Engineering Mathematics 9th Edition’.

JOHN BIRD

**Formerly Royal Naval Defence College of Marine
and Air Engineering, HMS Sultan,
University of Portsmouth
and Highbury College, Portsmouth**

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For Students

1. **Full solutions** to the 2100 questions contained in the 317 Practice Exercises
2. Revision of some important algebra topics
3. **List of Essential Formulae**
4. **Famous Engineers/Scientists** – 32 are mentioned in the text.
5. **Copies of chapters from the previous edition that have been excluded from this text** (these being: ‘Inequalities’, ‘Arithmetic and geometric progressions’, and ‘Binary, octal and hexadecimal numbers’)

For instructors/lecturers

1. **Full solutions** to the 2100 questions contained in the 317 Practice Exercises
2. **Full solutions** and marking scheme to each of the **20 Revision Tests**
3. **Revision Tests** – available to run off to be given to students
4. **List of Essential Formulae**
5. **Illustrations** – all 573 available on PowerPoint
6. **Famous Engineers/Scientists** – 32 are mentioned in the text

Syllabus guidance

This textbook is written for **undergraduate engineering degree and foundation degree courses**; however, it is also most appropriate for **HNC/D studies** and three syllabuses are covered. The appropriate chapters for these three syllabuses are shown in the table below.

Chapter		Analytical Methods for Engineers	Further Analytical Methods for Engineers	Advanced Mathematics
1.	Algebra	×		
2.	Partial fractions	×		
3.	Logarithms	×		
4.	Exponential functions	×		
5.	The binomial series	×		
6.	Solving equations by iterative methods		×	
7.	Boolean algebra and logic circuits		×	
8.	Introduction to trigonometry	×		
9.	Cartesian and polar co-ordinates	×		
10.	The circle and its properties	×		
11.	Trigonometric waveforms	×		
12.	Hyperbolic functions	×		
13.	Trigonometric identities and equations	×		
14.	The relationship between trigonometric and hyperbolic functions	×		
15.	Compound angles	×		
16.	Functions and their curves		×	
17.	Irregular areas, volumes and mean value of waveforms		×	
18.	Complex numbers		×	
19.	De Moivre's theorem		×	
20.	The theory of matrices and determinants		×	
21.	The solution of simultaneous equations by matrices and determinants		×	
22.	Vectors		×	
23.	Methods of adding alternating waveforms		×	
24.	Scalar and vector products		×	
25.	Methods of differentiation	×		

(Continued)

Chapter		Analytical Methods for Engineers	Further Analytical Methods for Engineers	Advanced Mathematics for Engineering
26.	Some applications of differentiation	×		
27.	Differentiation of parametric equations			
28.	Differentiation of implicit functions	×		
29.	Logarithmic differentiation	×		
30.	Differentiation of hyperbolic functions	×		
31.	Differentiation of inverse trigonometric and hyperbolic functions	×		
32.	Partial differentiation			×
33.	Total differential, rates of change and small changes			×
34.	Maxima, minima and saddle points for functions of two variables			×
35.	Standard integration	×		
36.	Some applications of integration	×		
37.	Maclaurin's series and limiting values	×		
38.	Integration using algebraic substitutions	×		
39.	Integration using trigonometric and hyperbolic substitutions	×		
40.	Integration using partial fractions	×		
41.	The $t = \tan \theta/2$ substitution			
42.	Integration by parts	×		
43.	Reduction formulae	×		
44.	Double and triple integrals			
45.	Numerical integration		×	
46.	Solution of first-order differential equations by separation of variables		×	
47.	Homogeneous first-order differential equations			
48.	Linear first-order differential equations		×	
49.	Numerical methods for first-order differential equations		×	×
50.	Second-order differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$		×	
51.	Second-order differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$		×	
52.	Power series methods of solving ordinary differential equations			×
53.	An introduction to partial differential equations			×

(Continued)

Chapter	Analytical Methods for Engineers	Further Analytical Methods for Engineers	Advanced Mathematics for Engineering
54.	Introduction to Laplace transforms		×
55.	Properties of Laplace transforms		×
56.	Inverse Laplace transforms		×
57.	The Laplace transform of the Heaviside function		
58.	Solution of differential equations using Laplace transforms		×
59.	The solution of simultaneous differential equations using Laplace transforms		×
60.	Fourier series for periodic functions of period 2π		×
61.	Fourier series for non-periodic functions over range 2π		×
62.	Even and odd functions and half-range Fourier series		×
63.	Fourier series over any range		×
64.	A numerical method of harmonic analysis		×
65.	The complex or exponential form of a Fourier series		×
66.	An introduction to z-transforms		
67.	Presentation of statistical data	×	
68.	Mean, median, mode and standard deviation	×	
69.	Probability	×	
70.	The binomial and Poisson distributions	×	
71.	The normal distribution	×	
72.	Linear correlation	×	
73.	Linear regression	×	
74.	Sampling and estimation theories	×	
75.	Significance testing	×	
76.	Chi-square and distribution-free tests	×	

Section A

Number and algebra



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Chapter 1

Algebra

Why it is important to understand: Algebra, polynomial division and the factor and remainder theorems

It is probably true to say that there is no branch of engineering, physics, economics, chemistry or computer science which does not require the understanding of the basic laws of algebra, the laws of indices, the manipulation of brackets, the ability to factorise and the laws of precedence. This then leads to the ability to solve simple, simultaneous and quadratic equations which occur so often. The study of algebra also revolves around using and manipulating polynomials. Polynomials are used in engineering, computer programming, software engineering, in management and in business. Mathematicians, statisticians and engineers of all sciences employ the use of polynomials to solve problems; among them are aerospace engineers, chemical engineers, civil engineers, electrical engineers, environmental engineers, industrial engineers, materials engineers, mechanical engineers and nuclear engineers. The factor and remainder theorems are also employed in engineering software and electronic mathematical applications, through which polynomials of higher degrees and longer arithmetic structures are divided without any complexity. The study of algebra, equations, polynomial division and the factor and remainder theorems is therefore of some considerable importance in engineering.

At the end of this chapter, you should be able to:

- understand and apply the laws of indices
- understand brackets, factorisation and precedence
- transpose formulae and solve simple, simultaneous and quadratic equations
- divide algebraic expressions using polynomial division
- factorise expressions using the factor theorem
- use the remainder theorem to factorise algebraic expressions

1.1 Introduction

In this chapter, polynomial division and the factor and remainder theorems are explained (in Sections 1.4 to 1.6). However, before this, some essential algebra revision on basic laws and equations is included.

For further algebra revision, go to the website:

www.routledge.com/cw/bird

1.2 Revision of basic laws

(a) Basic operations and laws of indices

The laws of indices are:

- (i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$
(iii) $(a^m)^n = a^{m \times n}$ (iv) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
(v) $a^{-n} = \frac{1}{a^n}$ (vi) $a^0 = 1$

4 Section A

Problem 1. Evaluate $4a^2bc^3 - 2ac$ when $a=2$, $b = \frac{1}{2}$ and $c = 1\frac{1}{2}$

$$\begin{aligned} 4a^2bc^3 - 2ac &= 4(2)^2 \left(\frac{1}{2}\right) \left(\frac{3}{2}\right)^3 - 2(2) \left(\frac{3}{2}\right) \\ &= \frac{4 \times 2 \times 2 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 2} - \frac{12}{2} \\ &= 27 - 6 = \mathbf{21} \end{aligned}$$

Problem 2. Multiply $3x + 2y$ by $x - y$

$$\begin{array}{r} 3x + 2y \\ \times \quad x - y \\ \hline \text{Multiply by } x \rightarrow 3x^2 + 2xy \\ \text{Multiply by } -y \rightarrow \quad -3xy - 2y^2 \\ \hline \text{Adding gives: } \quad \underline{3x^2 - xy - 2y^2} \end{array}$$

Alternatively,

$$\begin{aligned} (3x + 2y)(x - y) &= 3x^2 - 3xy + 2xy - 2y^2 \\ &= \mathbf{3x^2 - xy - 2y^2} \end{aligned}$$

Problem 3. Simplify $\frac{a^3b^2c^4}{abc^{-2}}$ and evaluate when $a = 3$, $b = \frac{1}{8}$ and $c = 2$

$$\frac{a^3b^2c^4}{abc^{-2}} = a^{3-1}b^{2-1}c^{4-(-2)} = \mathbf{a^2bc^6}$$

When $a = 3$, $b = \frac{1}{8}$ and $c = 2$,

$$a^2bc^6 = (3)^2 \left(\frac{1}{8}\right) (2)^6 = (9) \left(\frac{1}{8}\right) (64) = \mathbf{72}$$

Problem 4. Simplify $\frac{x^2y^3 + xy^2}{xy}$

$$\begin{aligned} \frac{x^2y^3 + xy^2}{xy} &= \frac{x^2y^3}{xy} + \frac{xy^2}{xy} \\ &= x^{2-1}y^{3-1} + x^{1-1}y^{2-1} \\ &= \mathbf{xy^2 + y} \quad \text{or} \quad \mathbf{y(xy + 1)} \end{aligned}$$

Problem 5. Simplify $\frac{(x^2\sqrt{y})(\sqrt{x}\sqrt[3]{y^2})}{(x^5y^3)^{\frac{1}{2}}}$

$$\begin{aligned} \frac{(x^2\sqrt{y})(\sqrt{x}\sqrt[3]{y^2})}{(x^5y^3)^{\frac{1}{2}}} &= \frac{x^2y^{\frac{1}{2}}x^{\frac{1}{2}}y^{\frac{2}{3}}}{x^{\frac{5}{2}}y^{\frac{3}{2}}} \\ &= x^{2+\frac{1}{2}-\frac{5}{2}}y^{\frac{1}{2}+\frac{2}{3}-\frac{3}{2}} \\ &= x^0y^{-\frac{1}{3}} \\ &= \mathbf{y^{-\frac{1}{3}}} \quad \text{or} \quad \mathbf{\frac{1}{y^{\frac{1}{3}}}} \quad \text{or} \quad \mathbf{\frac{1}{\sqrt[3]{y}}} \end{aligned}$$

Now try the following Practice Exercise

Practice Exercise 1 Basic algebraic operations and laws of indices (Answers on page 863)

- Evaluate $2ab + 3bc - abc$ when $a = 2$, $b = -2$ and $c = 4$
- Find the value of $5pq^2r^3$ when $p = \frac{2}{5}$, $q = -2$ and $r = -1$
- From $4x - 3y + 2z$ subtract $x + 2y - 3z$
- Multiply $2a - 5b + c$ by $3a + b$
- Simplify $(x^2y^3z)(x^3yz^2)$ and evaluate when $x = \frac{1}{2}$, $y = 2$ and $z = 3$
- Evaluate $(a^{\frac{3}{2}}bc^{-3})(a^{\frac{1}{2}}b^{-\frac{1}{2}}c)$ when $a = 3$, $b = 4$ and $c = 2$
- Simplify $\frac{a^2b + a^3b}{a^2b^2}$
- Simplify $\frac{(a^3b^{\frac{1}{2}}c^{-\frac{1}{2}})(ab)^{\frac{1}{3}}}{(\sqrt{a^3}\sqrt{bc})}$

(b) Brackets, factorisation and precedence

Problem 6. Simplify $a^2 - (2a - ab) - a(3b + a)$

$$\begin{aligned} a^2 - (2a - ab) - a(3b + a) \\ &= a^2 - 2a + ab - 3ab - a^2 \\ &= \mathbf{-2a - 2ab} \quad \text{or} \quad \mathbf{-2a(1 + b)} \end{aligned}$$

Problem 7. Remove the brackets and simplify the expression:

$$2a - [3\{2(4a - b) - 5(a + 2b)\} + 4a]$$

Removing the innermost brackets gives:

$$2a - [3\{8a - 2b - 5a - 10b\} + 4a]$$

Collecting together similar terms gives:

$$2a - [3\{3a - 12b\} + 4a]$$

Removing the 'curly' brackets gives:

$$2a - [9a - 36b + 4a]$$

Collecting together similar terms gives:

$$2a - [13a - 36b]$$

Removing the square brackets gives:

$$2a - 13a + 36b = -11a + 36b \quad \text{or} \\ 36b - 11a$$

Problem 8. Factorise (a) $xy - 3xz$
(b) $4a^2 + 16ab^3$ (c) $3a^2b - 6ab^2 + 15ab$

(a) $xy - 3xz = x(y - 3z)$

(b) $4a^2 + 16ab^3 = 4a(a + 4b^3)$

(c) $3a^2b - 6ab^2 + 15ab = 3ab(a - 2b + 5)$

Problem 9. Simplify $3c + 2c \times 4c + c \div 5c - 8c$

The order of precedence is division, multiplication, addition, and subtraction (sometimes remembered by BODMAS). Hence

$$\begin{aligned} 3c + 2c \times 4c + c \div 5c - 8c \\ &= 3c + 2c \times 4c + \left(\frac{c}{5c}\right) - 8c \\ &= 3c + 8c^2 + \frac{1}{5} - 8c \\ &= 8c^2 - 5c + \frac{1}{5} \quad \text{or} \quad c(8c - 5) + \frac{1}{5} \end{aligned}$$

Problem 10. Simplify $(2a - 3) \div 4a + 5 \times 6 - 3a$

$$\begin{aligned} (2a - 3) \div 4a + 5 \times 6 - 3a \\ &= \frac{2a - 3}{4a} + 5 \times 6 - 3a \\ &= \frac{2a - 3}{4a} + 30 - 3a \\ &= \frac{2a}{4a} - \frac{3}{4a} + 30 - 3a \\ &= \frac{1}{2} - \frac{3}{4a} + 30 - 3a = 30\frac{1}{2} - \frac{3}{4a} - 3a \end{aligned}$$

Now try the following Practice Exercise

Practice Exercise 2 Brackets, factorisation and precedence (Answers on page 863)

- Simplify $2(p + 3q - r) - 4(r - q + 2p) + p$
- Expand and simplify $(x + y)(x - 2y)$
- Remove the brackets and simplify:
 $24p - [2\{3(5p - q) - 2(p + 2q)\} + 3q]$
- Factorise $21a^2b^2 - 28ab$
- Factorise $2xy^2 + 6x^2y + 8x^3y$
- Simplify $2y + 4 \div 6y + 3 \times 4 - 5y$
- Simplify $3 \div y + 2 \div y - 1$
- Simplify $a^2 - 3ab \times 2a \div 6b + ab$

1.3 Revision of equations

(a) Simple equations

Problem 11. Solve $4 - 3x = 2x - 11$

Since $4 - 3x = 2x - 11$ then $4 + 11 = 2x + 3x$
i.e. $15 = 5x$ from which, $x = \frac{15}{5} = 3$

Problem 12. Solve

$$4(2a - 3) - 2(a - 4) = 3(a - 3) - 1$$

Removing the brackets gives:

$$8a - 12 - 2a + 8 = 3a - 9 - 1$$

Rearranging gives:

$$8a - 2a - 3a = -9 - 1 + 12 - 8$$

i.e. $3a = -6$

and $a = \frac{-6}{3} = -2$

Problem 13. The reaction moment M of a cantilever carrying three, point loads is given by:

$$M = 3.5x + 2.0(x - 1.8) + 4.2(x - 2.6)$$

where x is the length of the cantilever in metres. If $M = 55.32$ kN m, calculate the value of x .

If $M = 3.5x + 2.0(x - 1.8) + 4.2(x - 2.6)$ and

$$M = 55.32$$

then $55.32 = 3.5x + 2.0(x - 1.8) + 4.2(x - 2.6)$

i.e. $55.32 = 3.5x + 2.0x - 3.6 + 4.2x - 10.92$

and $55.32 + 3.6 + 10.92 = 9.7x$

i.e. $69.84 = 9.7x$

from which, **the length of the cantilever,**

$$x = \frac{69.84}{9.7} = 7.2 \text{ m}$$

Problem 14. Solve $\frac{3}{x-2} = \frac{4}{3x+4}$

By 'cross-multiplying': $3(3x+4) = 4(x-2)$

Removing brackets gives: $9x + 12 = 4x - 8$

Rearranging gives: $9x - 4x = -8 - 12$

i.e. $5x = -20$

and $x = \frac{-20}{5} = -4$

Problem 15. Solve $\left(\frac{\sqrt{t+3}}{\sqrt{t}}\right) = 2$

$$\sqrt{t} \left(\frac{\sqrt{t+3}}{\sqrt{t}}\right) = 2\sqrt{t}$$

i.e. $\sqrt{t+3} = 2\sqrt{t}$

and $3 = 2\sqrt{t} - \sqrt{t}$

i.e. $3 = \sqrt{t}$

and $9 = t$

(c) Transposition of formulae

Problem 16. Transpose the formula $v = u + \frac{ft}{m}$ to make f the subject.

$$u + \frac{ft}{m} = v \text{ from which, } \frac{ft}{m} = v - u$$

and $m \left(\frac{ft}{m}\right) = m(v - u)$

i.e. $ft = m(v - u)$

and $f = \frac{m}{t}(v - u)$

Problem 17. The impedance of an a.c. circuit is given by $Z = \sqrt{R^2 + X^2}$. Make the reactance X the subject.

$$\sqrt{R^2 + X^2} = Z \text{ and squaring both sides gives}$$

$$R^2 + X^2 = Z^2, \text{ from which,}$$

$$X^2 = Z^2 - R^2 \text{ and reactance } X = \sqrt{Z^2 - R^2}$$

Problem 18. Given that $\frac{D}{d} = \sqrt{\frac{f+p}{f-p}}$ express p in terms of D , d and f .

Rearranging gives: $\sqrt{\frac{f+p}{f-p}} = \frac{D}{d}$

Squaring both sides gives: $\frac{f+p}{f-p} = \frac{D^2}{d^2}$

'Cross-multiplying' gives:

$$d^2(f+p) = D^2(f-p)$$

Removing brackets gives:

$$d^2f + d^2p = D^2f - D^2p$$

Rearranging gives: $d^2p + D^2p = D^2f - d^2f$

Factorising gives: $p(d^2 + D^2) = f(D^2 - d^2)$

and

$$p = \frac{f(D^2 - d^2)}{(d^2 + D^2)}$$

Problem 19. Bernoulli's equation relates the flow velocity, v , the pressure, p , of the liquid and the height h of the liquid above some reference level.

Given two locations 1 and 2, the equation states:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

where ρ is the density of the liquid. Rearrange the equation to make the velocity of the liquid at location 2, i.e. v_2 , the subject.

Since
$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

then
$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 - \frac{p_2}{\rho g} - h_2 = \frac{v_2^2}{2g}$$

and
$$2g \left(\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 - \frac{p_2}{\rho g} - h_2 \right) = v_2^2$$

from which,
$$v_2 = \sqrt{2g \left(\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 - \frac{p_2}{\rho g} - h_2 \right)}$$

i.e. **the velocity at location 2,**

$$v_2 = \sqrt{2g \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g} + \frac{v_1^2}{2g} + h_1 - h_2 \right)}$$

Now try the following Practice Exercise

Practice Exercise 3 Simple equations and transposition of formulae (Answers on page 863)

In problems 1 to 4 solve the equations

- $3x - 2 - 5x = 2x - 4$
- $8 + 4(x - 1) - 5(x - 3) = 2(5 - 2x)$
- $\frac{1}{3a - 2} + \frac{1}{5a + 3} = 0$
- $\frac{3\sqrt{t}}{1 - \sqrt{t}} = -6$
- Transpose $y = \frac{3(F - f)}{L}$ for f
- Make l the subject of $t = 2\pi\sqrt{\frac{l}{g}}$
- Transpose $m = \frac{\mu L}{L + rCR}$ for L
- Make r the subject of the formula
$$\frac{x}{y} = \frac{1 + r^2}{1 - r^2}$$

9. Young's modulus, $E = \frac{\text{stress, } \sigma}{\text{strain, } \varepsilon}$ and
$$\text{strain, } \varepsilon = \frac{\text{length increase, } \delta}{\text{length, } \ell}$$

For a 50 mm length of a steel bolt under tensile stress, its length increases by 1.25×10^{-4} m. If $E = 2 \times 10^{11}$ N/m², calculate the stress.

10. The mass moment of inertia through the centre of gravity for a compound pendulum, I_G , is given by: $I_G = mk_G^2$. The value of the radius of gyration about G , k_G^2 , may be determined from the frequency, f , of a compound pendulum, given by:
$$f = \frac{1}{2\pi} \sqrt{\frac{gh}{(k_G^2 + h^2)}}$$

Given that the distance $h = 50$ mm, $g = 9.81$ m/s² and the frequency of oscillation, $f = 1.26$ Hz, calculate the mass moment of inertia I_G when $m = 10.5$ kg.

(d) Simultaneous equations

Problem 20. Solve the simultaneous equations:

$$7x - 2y = 26 \quad (1)$$

$$6x + 5y = 29 \quad (2)$$

$5 \times$ equation (1) gives:

$$35x - 10y = 130 \quad (3)$$

$2 \times$ equation (2) gives:

$$12x + 10y = 58 \quad (4)$$

Equation (3) + equation (4) gives:

$$47x + 0 = 188$$

from which,
$$x = \frac{188}{47} = 4$$

Substituting $x = 4$ in equation (1) gives:

$$28 - 2y = 26$$

from which, $28 - 26 = 2y$ and $y = 1$

Problem 21. Solve

$$\frac{x}{8} + \frac{5}{2} = y \quad (1)$$

$$11 + \frac{y}{3} = 3x \quad (2)$$

8 Section A

$$8 \times \text{equation (1) gives: } x + 20 = 8y \quad (3)$$

$$3 \times \text{equation (2) gives: } 33 + y = 9x \quad (4)$$

$$\text{i.e. } x - 8y = -20 \quad (5)$$

$$\text{and } 9x - y = 33 \quad (6)$$

$$8 \times \text{equation (6) gives: } 72x - 8y = 264 \quad (7)$$

Equation (7) – equation (5) gives:

$$71x = 284$$

$$\text{from which, } x = \frac{284}{71} = 4$$

Substituting $x = 4$ in equation (5) gives:

$$4 - 8y = -20$$

from which, $4 + 20 = 8y$ and $y = 3$

(e) Quadratic equations

Problem 22. Solve the following equations by factorisation:

(a) $3x^2 - 11x - 4 = 0$

(b) $4x^2 + 8x + 3 = 0$

(a) The factors of $3x^2$ are $3x$ and x and these are placed in brackets thus:

$$(3x \quad \quad)(x \quad \quad)$$

The factors of -4 are $+1$ and -4 or -1 and $+4$, or -2 and $+2$. Remembering that the product of the two inner terms added to the product of the two outer terms must equal $-11x$, the only combination to give this is $+1$ and -4 , i.e.,

$$3x^2 - 11x - 4 = (3x + 1)(x - 4)$$

Thus $(3x + 1)(x - 4) = 0$ hence

either $(3x + 1) = 0$ i.e. $x = -\frac{1}{3}$

or $(x - 4) = 0$ i.e. $x = 4$

(b) $4x^2 + 8x + 3 = (2x + 3)(2x + 1)$

Thus $(2x + 3)(2x + 1) = 0$ hence

either $(2x + 3) = 0$ i.e. $x = -\frac{3}{2}$

or $(2x + 1) = 0$ i.e. $x = -\frac{1}{2}$

Problem 23. The roots of a quadratic equation are $\frac{1}{3}$ and -2 . Determine the equation in x .

If $\frac{1}{3}$ and -2 are the roots of a quadratic equation then,

$$(x - \frac{1}{3})(x + 2) = 0$$

i.e. $x^2 + 2x - \frac{1}{3}x - \frac{2}{3} = 0$

i.e. $x^2 + \frac{5}{3}x - \frac{2}{3} = 0$

or $3x^2 + 5x - 2 = 0$

Problem 24. The stress, ρ , set up in a bar of length, ℓ , cross-sectional area, A , by a mass W , falling a distance h is given by the formula:

$$\rho^2 - \frac{2W}{A}\rho - \frac{2WEh}{A\ell} = 0$$

Given that $E = 12500$, $\ell = 110$, $h = 0.5$, $W = 2.25$ and $A = 3.20$, calculate the positive value of ρ correct to 3 significant figures.

Since $\rho^2 - \frac{2W}{A}\rho - \frac{2WEh}{A\ell} = 0$ and $E = 12500$, $\ell = 110$, $h = 0.5$, $W = 2.25$ and $A = 3.20$ then

$$\rho^2 - \frac{2(2.25)}{3.20}\rho - \frac{2(2.25)(12500)(0.5)}{(3.20)(110)} = 0$$

i.e.

$$\rho^2 - 1.40625\rho - 79.90057 = 0$$

Solving using the quadratic formula gives:

$$\rho = \frac{-(-1.40625) \pm \sqrt{(-1.40625)^2 - 4(1)(-79.90057)}}{2(1)}$$

$$= 9.669 \text{ or } -8.263$$

Hence, correct to 3 significant figures, **the positive value of ρ is 9.67**

Now try the following Practice Exercise

Practice Exercise 4 Simultaneous and quadratic equations (Answers on page 863)

In problems 1 to 3, solve the simultaneous equations

1. $8x - 3y = 51$

$$3x + 4y = 14$$

2. $5a = 1 - 3b$

$2b + a + 4 = 0$

3. $\frac{x}{5} + \frac{2y}{3} = \frac{49}{15}$

$\frac{3x}{7} - \frac{y}{2} + \frac{5}{7} = 0$

4. In an engineering scenario involving reciprocal motion, the following simultaneous equations resulted: $\omega\sqrt{(r^2 - 0.07^2)} = 8$ and $\omega\sqrt{(r^2 - 0.25^2)} = 2$. Calculate the value of radius r and angular velocity ω , each correct to 3 significant figures.

5. Solve the following quadratic equations by factorisation:

(a) $x^2 + 4x - 32 = 0$

(b) $8x^2 + 2x - 15 = 0$

6. Determine the quadratic equation in x whose roots are 2 and -5

7. Solve the following quadratic equations, correct to 3 decimal places:

(a) $2x^2 + 5x - 4 = 0$

(b) $4t^2 - 11t + 3 = 0$

8. A point of contraflexure from the left-hand end of a 6 m beam is given by the value of x in the following equation:

$$-x^2 + 11.25x - 22.5 = 0$$

Determine the point of contraflexure.

9. The vertical height, h , and the horizontal distance travelled, x , of a projectile fired at an angle of 45° at an initial velocity, v_0 , are related by the equation:

$$h = x - \frac{gx^2}{v_0^2}$$

If the projectile has an initial velocity of 120 m/s, calculate the values of x when the projectile is at a height of 200 m, assuming that $g = 9.81 \text{ m/s}^2$. Give the answers correct to 3 significant figures.

1.4 Polynomial division

Before looking at long division in algebra let us revise long division with numbers (we may have forgotten, since calculators do the job for us!).

For example, $\frac{208}{16}$ is achieved as follows:

$$\begin{array}{r} 13 \\ 16 \overline{) 208} \\ \underline{16} \\ 48 \\ \underline{48} \\ 0 \end{array}$$

- (1) 16 divided into 2 won't go
- (2) 16 divided into 20 goes 1
- (3) Put 1 above the zero
- (4) Multiply 16 by 1 giving 16
- (5) Subtract 16 from 20 giving 4
- (6) Bring down the 8
- (7) 16 divided into 48 goes 3 times
- (8) Put the 3 above the 8
- (9) $3 \times 16 = 48$
- (10) $48 - 48 = 0$

Hence $\frac{208}{16} = 13$ exactly

Similarly, $\frac{172}{15}$ is laid out as follows:

$$\begin{array}{r} 11 \\ 15 \overline{) 172} \\ \underline{15} \\ 22 \\ \underline{15} \\ 7 \end{array}$$

Hence $\frac{172}{15} = 11$ remainder 7 or $11 + \frac{7}{15} = 11\frac{7}{15}$

Below are some examples of division in algebra, which in some respects is similar to long division with numbers.

(Note that a **polynomial** is an expression of the form

$$f(x) = a + bx + cx^2 + dx^3 + \dots$$